

**Goals:**

1. Determine the inverse of a given function
2. Determine whether two functions are inverses
3. Graph the inverse of a function



Two functions are inverse functions if one function “undoes” what the other function “does”.

Inverse functions are just a reflection across the line  $y = x$ . In order for both a function  $f(x)$  and its inverse  $f^{-1}(x)$  to qualify as functions,  $f(x)$  must be a one-to-one function.

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**Determining the inverse of a function:**

1. Verify that  $f(x)$  is a one-to-one function. (If not, its inverse is not a function.)
2. Replace  $f(x)$  with  $y$  and exchange all  $x$ 's and  $y$ 's.
3. Solve for  $y$ .
4. Replace the new  $y$  with  $f^{-1}(x)$ . (Only if  $f^{-1}(x)$  is actually a function.)

**Example 1:** If  $f(x) = 3x - 2$ , determine the inverse of the function.

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**Determining if two functions are inverses:** (‘undo’ each other)

Two functions  $f(x)$  and  $g(x)$  are inverses of each other **if and only if**

$$(f \circ g)(x) = x \text{ for every value of } x \text{ in the domain of } g \text{ and}$$

$$(g \circ f)(x) = x \text{ for every value of } x \text{ in the domain of } f.$$


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**Example 2:** Determine whether  $f(x) = \frac{x}{2x-3}$  and  $g(x) = \frac{3x}{2x-1}$  are inverses of each other.

**Graphing the inverse of a function**

The graph of a function and its inverse are symmetric about the line  $y = x$ .

$f(x)$  is the reflection of  $f^{-1}(x)$  on the line  $y = x$  and vice versa.

**Example 3:** Given the function  $f(x) = (x+2)^2 + 4$ , graph the inverse of the function. Determine whether the inverse of the function is a function.

