Warmup - The starting salary of an employee is $21,250. If a raise of $1,250 is given each year, in how many years will the employee’s salary be $50,000?

The problem above can be solved using a formula, which we will develop:

A sequence is simply an ordered list of numbers (called terms) that follow a pattern so that the next term can be determined.

- The first term in the sequence is labeled $t_1$.
- The number of terms in the sequence is $n$.
- Any term of the sequence is $t_n$ (read $t$ sub $n$), dependent on the value of $n$. For example, the third term is $t_3$, the eighth term is $t_8$ etc.
- A finite sequence has a finite number of terms whereas an infinite sequence has an infinite number of terms.
- An arithmetic sequence is an ordered list of terms in which the difference between consecutive terms is constant (a common difference $d$).

For the problem above, what is $t_1$? What is $d$?

$$t_1 = \quad \quad d =$$

How can you find $t_2$ using $t_1$ and $d$? \quad $t_2 = \quad \quad$ in general: $t_2 =$

How can you find $t_3$ using $t_1$ and $d$? \quad $t_3 = \quad \quad$ in general: $t_3 =$

How can you find $t_4$ using $t_1$ and $d$? \quad $t_4 = \quad \quad$ in general: $t_4 =$

Can you develop a formula for the general term of an arithmetic sequence?

Use the formula to answer the problem from the top of the page:
Example – Jim had 35 baseball cards in 2016. Every year, he adds 9 cards to his collection. How many cards will he have at the end of 2027?

Example – The growth of a colony of carpenter ants produces an arithmetic sequence in which the number of ants increases by approximately 80 ants each month. Beginning with 40 ants, how many months would it take for the ant population to reach 3000?
Example – A furnace technician charges $45 for making a house call, plus $46 per hour or portion of an hour.

a) Generate the possible charges for the first 4 hours of time.
b) What is the charge for 10 hours of time?

Example – The fourth term of an arithmetic sequence is -10. The eleventh term is 32. Determine $d$, $t_1$, and write the first six terms of the sequence.
1.2 – Arithmetic Series

Key Ideas:

**Gauss' Method**

Warmup – Read the top half of p.22. Can you answer the question of what Gauss did next?

Use Gauss’ Method to calculate the sum of the numbers from 1 to 8 (show your work).

Building off of the example above, we’ll derive a general formula for the sum of an arithmetic series using $S_n$ as the sum of an arithmetic series, $t_1$ as the first term, $n$ as the number of terms, and $d$ as the common difference.

<table>
<thead>
<tr>
<th>What does the 100 represent in Gauss’ formula?</th>
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<td>What does the 101 represent?</td>
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| Write a general formula for Gauss’ method:

<table>
<thead>
<tr>
<th>The sum of an arithmetic series can be determined using the formula</th>
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<td>where $t_1 = \text{first term}$, $t_n = \text{last term}$, $n = \text{number of terms}$, and $S_n = \text{sum of the first } n \text{ terms}$</td>
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From section 1.1, $t_n = t_1 + (n - 1)d$, so the formula for $S_n$ can be written another way.

<table>
<thead>
<tr>
<th>The sum of an arithmetic series can also be determined using the formula:</th>
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Example – Fireflies flash in patterns to signal location or ward off predators. Suppose a firefly flashes twice in the first minute, four times in the second minute, and six times in the third minute.

a) If this pattern continues, what is the number of flashes in the 42nd minute?

b) What is the total number of flashes for the male firefly after 42 minutes?

Example – The sum of the first two terms of an arithmetic series is 19 and the sum of the first four terms is 50. What are the first six terms of the series and the sum to 20 terms?

Example – For the arithmetic series, determine the value of $n$: $t_1 = -6$, $t_n = 21$, $S_n = 75$. Then, for the following series, determine $t_2$: $d = 0.5$, $S_n = 218.5$, $n = 23$. 

using the correct formula
A geometric sequence is a sequence in which the ratio of consecutive terms is constant.

Warmup – Suppose you have the geometric sequence 4, 12, 36, 108, ...

a) What is $t_1$?
b) What do you multiply by to get the next term (this is the $r$ value)?
c) Is the sequence geometric (see the definition above)? In other words, is the $r$ value consistent throughout the sequence?
d) What is $t_5$? Explain how you got $t_5$. Write a general formula for this.

e) Show how to get $t_5$ using only $t_1$ and $r$.

f) Show how to get $t_8$ using only $t_1$ and $r$.

g) What do you notice about the exponent on $r$ compared to $n$?

h) Write a general formula for $t_n$ for any geometric sequence:

The general term of a geometric sequence where $n$ is a positive integer is:

\[ t_n = t_1 \cdot r^{(n-1)} \]

OR

where $t_1$ is the first term, $n$ is the number of terms, $r$ is the common ratio, and $t_n$ is a general term

For a geometric sequence, the common ratio ($r$), can be found by taking any term (except the first) and dividing that term by the preceding term. So $r = \frac{t_n}{t_{n-1}}$

Example – Are the following sequences geometric (ie. Is the $r$ value consistent)?

a) 2, 4, 6, 8

b) 4, 10, 25, 62.5
Example – Bacteria reproduce by splitting into two. Suppose there were three bacteria originally present in a sample. How many bacteria will there be after 8 generations?

Example – Suppose a photocopier can reduce a picture to 60% of its original size. If the picture is originally 42cm long, what length will it be after five successive reductions?

Example – In a geometric sequence, the second term is 28 and the fifth term is 1792. Determine the values of \( t_1 \) and \( r \), and list the first three terms of the sequence.

Example – In 1990 the population of Canada was approximately 26.6 million. The population projection for 2025 is approximately 38.4 million. If this projection were based on a geometric sequence, what would be the annual growth rate?

Percentages

If a question involves percent growth, \( r \) must be greater than 1.

Ex. If there is 30% growth each year, what is the \( r \) value for the problem?

If a question involves a percent reduction, \( r \) must be less than 1 and must represent the percent remaining (not the percent lost).

Ex. If you reduce the size of your savings by 25% per year, what is \( r \)?
One example of a geometric series is a phone tree. Suppose the school band had to share information that they didn’t want public, so they used a phone tree. Draw the first four levels of the tree below:

What pattern has developed?

What is the common ratio of the sequence?

Pages 48 & 49 of the textbook will take you through a derivation of the formulas for a Geometric Series. It is a challenging derivation, thus we will not do it as a class. Below are the developed formulas:

The sum of a geometric series can be determined using the formula

$$S_n = \frac{t_1(r^n-1)}{r-1}, r \neq 1$$

OR

$$S_n = \frac{r t_n - t_1}{r-1}, r \neq 1$$ (see p.51)

where $t_1$ is the first term, $n$ is the number of terms, $r$ is the common ratio, and $S_n$ is the sum of the first $n$ terms

Use the formula(s) to find the amount of people reached in the school band after 7 layers of the phone tree:
Example – Determine the sum of the first 8 terms of the following geometric series.

a) \(5 + 15 + 45 + \ldots\)

b) \(t_2 = 64, \ r = \frac{1}{4}\)

Example – Determine the sum of the following geometric series.

a) \(\frac{1}{64} + \frac{1}{16} + \frac{1}{4} + \ldots + 1024\)

b) \(-2 + 4 - 8 + \ldots - 8192\)

Example – A scrabble tournament with 512 players is held. When a player loses, he/she is eliminated. The winners continue to play until a final match determines the champion. What is the total number of matches that will be played in the tournament?
An **infinite geometric series** is a geometric series that...

#1) Consider the infinite geometric series $1 + 2 + 4 + 8 + 16 + ...$ What would the sum be? What is the $r$ value?

#2) Consider the infinite geometric series that has $t_1 = 4$ and $r = \frac{1}{2}$. Write the series up to 13 terms and find the sum for $S_5, S_7, S_9, S_{11}, \& S_{13}$.

When the sum approaches a fixed value, the series is said to be **convergent**. When this is the case, $r$ must be between -1 and 1.

If, in an infinite series, each term continues to grow, the sum does not approach a fixed value. It actually approaches infinity or negative infinity. In these situations, $r$ is less than -1 or greater than 1. The infinite series is said to be **divergent**.

For infinite series that are convergent, the formula for finding the sum that the series converges to is

$$S_\infty = \frac{t_1}{1-r}$$

where $t_1$ is the first term, $r$ is the common ratio, and $S_\infty$ is the sum of an infinite number of terms.

Use the formula to find the sum of the infinite series from #2 above:
Example – Determine whether each infinite geometric series converges or diverges. Calculate the sum.

a) \( 1 + \frac{1}{5} + \frac{1}{25} + \cdots \)

b) \( 4 - 8 + 16 - 32 + \cdots \)

Example - If the first term of an infinite geometric series is 12, and the sum is 48. determine \( r \).