Elastic Collisions Key

40. (a) Using the coordinate system shown, for momentum conservation we have

x-momentum: $mv + 0 = 0 + 2mv_2' \cos\theta$, or
$2v_2' \cos\theta = v$;

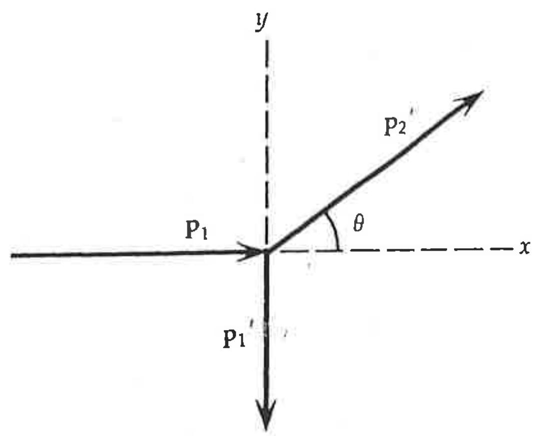
y-momentum: $0 + 0 = -mv_1' + 2mv_2' \sin\theta$, or
$2v_2' \sin\theta = v_1'$.

If we square and add these two equations, we get
$v^2 + v_1'^2 = 4v_2'^2$.

For the conservation of kinetic energy, we have
$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_1'^2 + \frac{1}{2}(2m)v_2'^2$, or
$v^2 - v_1'^2 = 2v_2'^2$.

When we add this to the previous result, we get
$v^2 = 3v_2'^2$.

Using this in the x-momentum equation, we get
$\cos\theta = \sqrt{3}/2$, or $\theta = \boxed{30°.}$



(b) From part (a) we have
$v_2' = v/\sqrt{3}$.

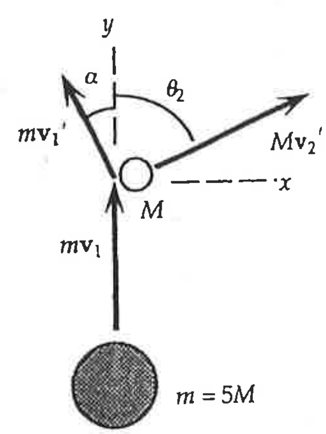
Using the energy result, we get
$v_1'^2 = v^2 - 2v_2'^2 = v^2 - 2v^2/3 = \frac{1}{3}v^2$, or $\boxed{v_1' = v/\sqrt{3}.}$

(c) The fraction of the kinetic energy transferred is
fraction $= KE_2/KE_1 = \frac{1}{2}(2m)v_2'^2/\frac{1}{2}mv^2 = m(v^2/3)/\frac{1}{2}mv^2 = \boxed{\frac{2}{3}.}$

42. Using the coordinate system shown, for momentum conservation we have

y-momentum: $mv_1 + 0 = mv_1' \cos\alpha + Mv_2' \cos\theta_2$;
$5M(12.0\text{ m/s}) = 5Mv_1' \cos\alpha + Mv_2' \cos 80°$, or
$5v_1' \cos\alpha = -v_2' \cos 80° + 60.0\text{ m/s}$.

x-momentum: $0 = -mv_1' \sin\alpha + Mv_2' \sin\theta_2$;
$0 = -5Mv_1' \sin\alpha + Mv_2' \sin 80°$, or
$5v_1' \sin\alpha = v_2' \sin 80°$.



For the conservation of kinetic energy, we have
$\frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv_1'^2 + \frac{1}{2}Mv_2'^2$;
$5M(12.0\text{ m/s})^2 = 5Mv_1'^2 + Mv_2'^2$, or
$5v_1'^2 + v_2'^2 = 720\text{ m}^2/\text{s}^2$.

We have three equations in three unknowns: α, v_1', v_2'. We eliminate α by squaring and adding the two momentum results, and then combine this with the energy equation, with the results:

(a) $v_2' = \boxed{3.47\text{ m/s.}}$

(b) $v_1' = \boxed{11.9\text{ m/s.}}$

(c) $\alpha = \boxed{3.29°.}$

#42 $\left(5V_1' \cos\alpha = -V_2' \cos 80° + 60\right)$ ①

$\left(5V_1' \sin\alpha = V_2' \sin 80°\right)$ ②

SQUARING ① + ②

$$25V_1'^2 \cos^2\alpha = 3600 - 120V_2' \cos 80° + V_2'^2 \cos^2 80°$$

$$25V_1'^2 \sin^2\alpha = V_2'^2 \sin^2 80°$$

add

$$25V_1'^2\left(\cancel{\cos^2\alpha + \sin^2\alpha}\right) = 3600 - 120V_2'\cos 80° + V_2'^2\left(\cancel{\cos^2 80° + \sin^2 80°}\right)$$

$$25V_1'^2 = 3600 - 120V_2'\cos 80° + V_2'^2$$

$$V_1'^2 = \left(\frac{V_2'^2 - 120V_2'\cos 80° + 3600}{25}\right)$$

SUB INTO $\quad 5V_1'^2 + V_2'^2 = 720$

$$5\left(\frac{V_2'^2 - 120V_2'\cos 80° + 3600}{25}\right) + V_2'^2 = 720$$

$$5V_2'^2 - 600V_2'\cos 80° + 18000 + 25V_2'^2 = 18000$$

$$30V_2'^2 - V_2'600\cos 80° = 0$$

$$30V_2'^2 = V_2'600\cos 80°$$

$$\therefore V_2' = \frac{600\cos 80°}{30}$$

$$V_2' = 3.47296 \text{ m/s}$$

$$5V_1'^2 + V_2'^2 = 720$$

$$\therefore \quad V_1' = \sqrt{\frac{720 - V_2'^2}{5}}$$

$$V_1' = \sqrt{\frac{720 - (3.47296)^2}{5}} = \underline{\underline{11.8999}} \text{ m/s}$$

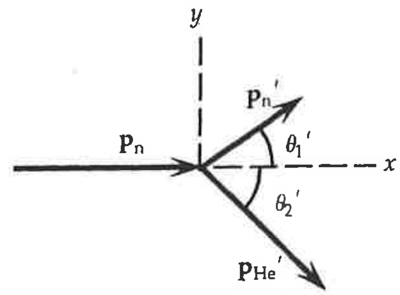
$$5V_1' \sin \alpha = V_2' \sin 80^\circ$$

$$\sin \alpha = \frac{V_2' \sin 80^\circ}{5V_1'}$$

$$\alpha = \sin^{-1}\left(\frac{(3.47296)(\sin 80^\circ)}{5(11.899)}\right)$$

$$\alpha = \underline{\underline{3.29^\circ}}$$

43. Using the coordinate system shown, for momentum conservation we have



x: $m_n v_n + 0 = m_n v_n' \cos \theta_1' + m_{He} v_{He}' \cos \theta_2'$;
$m_n(6.2 \times 10^5 \text{ m/s}) = m_n v_n' \cos \theta_1' + 4m_n v_{He}' \cos 45°$, or
$v_n' \cos \theta_1' = (6.2 \times 10^5 \text{ m/s}) - 4v_{He}' \cos 45°$.

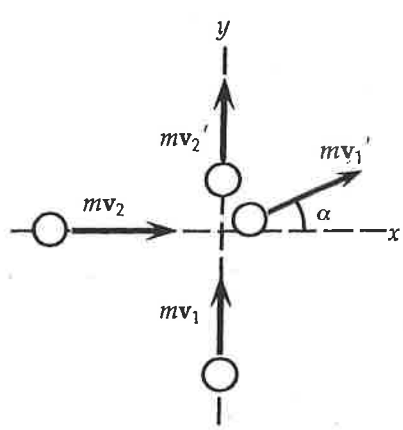
y: $0 + 0 = -m_n v_n' \sin \theta_1' + m_{He} v_{He}' \sin \theta_2'$;
$0 = -m_n v_n' \sin \theta_1' + 4m_n v_{He}' \sin 45°$, or
$v_n' \sin \theta_1' = 4v_{He}' \sin 45°$.

For the conservation of kinetic energy, we have
$\frac{1}{2}m_n v_n^2 + 0 = \frac{1}{2}m_n v_n'^2 + \frac{1}{2}m_{He} v_{He}'^2$;
$m_n(6.2 \times 10^5 \text{ m/s})^2 = m_n v_n'^2 + 4m_n v_{He}'^2$, or
$v_n'^2 + 4v_{He}'^2 = 3.84 \times 10^{11} \text{ m}^2/\text{s}^2$.

We have three equations in three unknowns: θ_1', v_n', v_{He}'. We eliminate θ_1' by squaring and adding the two momentum results, and then combine this with the energy equation, with the results:

$\boxed{\theta_1' = 76°, v_n' = 5.1 \times 10^5 \text{ m/s}, v_{He}' = 1.8 \times 10^5 \text{ m/s}.}$

44. Using the coordinate system shown, for momentum conservation we have



x: $0 + mv_2 = mv_1' \cos \alpha + 0$;
$3.7 \text{ m/s} = v_1' \cos \alpha$;

y: $mv_1 + 0 = mv_1' \sin \alpha + mv_2'$;
$2.0 \text{ m/s} = v_1' \sin \alpha + v_2'$, or
$v_1' \sin \alpha = 2.0 \text{ m/s} - v_2'$.

For the conservation of kinetic energy, we have
$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2$;
$(2.0 \text{ m/s})^2 + (3.7 \text{ m/s})^2 = v_1'^2 + v_2'^2$.

We have three equations in three unknowns: α, v_1', v_2'. We eliminate α by squaring and adding the two momentum results, and then combine this with the energy equation, with the results:

$\boxed{\alpha = 0°, v_1' = 3.7 \text{ m/s}, v_2' = 2.0 \text{ m/s}.}$

The two billiard balls exchange velocities.